

Modulation of ion-acoustic waves in a nonextensive plasma with two-temperature electrons

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We study the amplitude modulation of ion-acoustic wave (IAW) packets in an unmagnetized electron-ion plasma with two-temperature (cool and hot) electrons in the context of the Tsallis' nonextensive statistics. Using the multiple-scale technique, a nonlinear Schrödinger (NLS) equation is derived which governs the dynamics of modulated wave packets. It is shown that in nonextensive plasmas, the IAW envelope is always stable for long-wavelength modes ($k \rightarrow 0$) and unstable for short-wavelengths with $0 < k < 1$. However, the envelope can be unstable at an intermediate scale of perturbations with $0 < k < 1$. Thus, the modulated IAW packets can propagate in the form of bright envelope solitons or rogons (at small- and medium scale perturbations) as well as dark envelope solitons (at large scale). The stable and unstable regions are obtained for different values of temperature and density ratios as well as the nonextensive parameters q_c and q_h for cool and hot electrons. It is found that the more (less) the population of superthermal cool (hot) electrons, the smaller is the growth rate of instability with cutoffs at smaller wave numbers of modulation.

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I. INTRODUCTION

Over the last few decades, wave dynamics in multi-component plasmas has been a front area of research for the plasma community. Various types of nonlinear waves namely, ion-acoustic waves (IAWs), electron-acoustic waves (EAWs) etc. as well as nonlinear coherent structures associated with them namely, solitons, envelope solitons, shocks etc. have also been studied in different plasma environments. As is known, ion-acoustic waves are electrostatic plasma modes, where a population of inertial ions oscillate against a dominant thermalized background of electrons that provide necessary restoring force. The phase velocity of such waves lies between the electron and ion thermal velocities. Plasmas with two groups of electrons (e.g., cool and hot electrons) are very common in laboratories [1–3] as well as in space environments [4]. In this context, a large number of investigations have been made to study the characteristics of IAWs in two-temperature electron plasmas [5–9]. Such two-temperature plasmas have been observed by various satellite missions, e.g., Fast Auroral Snapshot (FAST) at the auroral region [10], Viking Satellite [4] and THEMIS mission [11]. Furthermore, several investigations made by GEOTAIL and POLAR [12] in the magnetosphere have also confirmed the existence of such electron populations. Jones et al. [5] reported experimentally the existence of ion acoustic solitary waves (IASWs) in plasmas with hot and cold electrons. Baluku et al. [13] studied

small-amplitude ion-acoustic solitons in a plasma with two-temperature kappa distributed electrons and found that superthermality of electrons played an important role for the modification of solitary structures.

The linear properties of the IAWs have been extensively studied and well understood in multi-component plasmas in presence of hot electrons obeying Maxwellian and non-Maxwellian velocity distributions. When the nonlinear effects are concerned, the formation of ion-acoustic solitons may be possible due to a delicate balance between the nonlinearity and the dispersion which also has been anticipated theoretically via the derivation of Korteweg-de Vries (KdV) or Zakharov-Kuznetsov (ZK) equations for small-amplitude perturbations [14, 15], and via the Sagdeev potential formalism, accounting for the arbitrary amplitude excitations [16]. Existence of these nonlinear structures was experimentally confirmed [17].

The space observations and laboratory experiments indicate the presence of particles which obey non-Maxwellian velocity distributions in plasmas. The observations of the voyager PLS [18, 19] have also shown the existence of both cool and hot electrons populations (non-Maxwellian) in Saturn's Magnetosphere. One of such distributions as reported by Renyi [20] and proposed by Tsallis [21] is the Boltzmann-Gibbs-Shannon (BGS) entropy in which the degree of nonextensivity of the plasma particles is characterized by the entropic index q , i.e., the distribution function with $q < 1$, compared with the Maxwellian one ($q = 1$) indicates the system with more superthermal particles (superextensivity), whereas the q distribution with $q > 1$ is suitable for plasmas containing a large number low-speed particles (subextensivity). However, because of long-range Coulomb force between plasma particles and the

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presence of many superthermal particles in astrophysical and space environments a q -distribution with $q < 1$ is strongly suggested for real plasmas or superthermal plasmas. Such q -nonextensive distribution has been widely used to study various nonlinear coherent structures like solitons, double layers or shocks etc. [22–28].

The propagation of wave packets in a nonlinear, dispersive medium is subjected to the modulation of their wave amplitudes, i.e., a slow variation of the wave packets envelope due to the nonlinear self-interaction of the carrier wave modes. The systems evolution is then governed through the nonlinear Schrödinger (NLS) equation and the associated modulational instability (MI). The latter in nonlinear, dispersive media has been a well-known mechanism for the localization of wave energy. Furthermore, such instability results into the exponential growth of a small plane wave perturbation which leads to the amplification of the sidebands, and thereby breaking up the uniform wave into a train of oscillations. Thus, the MI of wave packets in plasmas acts as a precursor for the formation of bright envelope solitons or highly energetic rogue waves (rogons), otherwise the dark envelope solitons may be formed. The amplitude modulation of electrostatic waves in plasmas have been investigated by a number of authors owing to their importance in space, astrophysical and laboratory plasmas. A great deal of attention has been given to the MI of IAWs in non-Maxwellian plasmas. To mention few, Saini and Kourakis [29] investigated the amplitude modulation of dust-acoustic waves in dusty plasmas with superthermal ions. Similar investigations have also been made of IAWs in plasmas with two-temperature electrons [30] and kappa-distributed superthermal electrons [31]. Furthermore, Bains et al. [25] studied the MI of ion-acoustic wave envelopes in a plasma containing non-extensive electrons.

On the other hand, the generation of highly energetic rogue waves (or rogons) in plasmas has been a topic of important research. Such waves may also appear in other environments such as oceanic rogue waves [32, 33], stock market crashes [34], optical rogue waves [35, 36], superfluid helium [37], Bose-Einstein condensates [38], atmospheric physics [39] and opposing current flow [40]. The rogue waves in plasmas may be generated due to wave-wave interactions [41], and their evolution can also be described by the NLS equation for modulated wave packets. The rogue waves were first observed in a ocean [42]. The space-time localized solution (rational solution) of the NLS equation was determined and later recognized as Peregrine solitons (a prototype of rogue waves). The latter were also experimentally observed in different nonlinear dispersive media (e.g., optical fiber, plasma etc.). The possible mechanism to explain these waves include effects such as nonlinear focusing via MI in one space dimension. For some theoretical description of rogue waves readers are referred to Ref.43 and some experimental observations to Refs.36, 44–46, 49.

We, however, mention that the physical mechanism behind the formation of rogue waves is still unclear,

however, observations indicate that they have unusually steep, solitary or tightly grouped profiles, which appear like “walls of water” [41]. A large number of theoretical and experimental investigations have been reported for the study of rogue waves in various plasma systems [45, 50–60]. Abdelsalam [58] studied the nonlinear propagation of solitary and freak waves in a superthermal plasma with positive and negative ions, ion beam and stationary dust particles. More recently, Shalini and Saini [61] have considered the propagation of DIA rogue waves in superthermal plasmas and investigated the combined effects of superthermality and dust concentration on the characteristics of DIA rogue waves.

In this work, we consider the amplitude modulation of IAW packets in an unmagnetized plasma whose constituents are inertial cold positive ions and two-temperature (cool and hot) electrons obeying Tsallis’ nonextensive q distribution. The nonlinear evolution of these waves are governed by a NLS equation which is used to study the MI of a Stokes’ wave train to a small longitudinal perturbation. It is shown that the wave packets with long-wavelength modes ($k \rightarrow 0$) are always stable, whereas modes with $k \gtrsim 1$ gives rise the MI of IAWs. The manuscript is organized as follows: In Sec. II, we present the fluid model for IAWs. Using the multiple scale perturbation technique, the expressions for different order harmonic modes along with the NLS equation are presented in Sec. III. Section IV describes the criteria for MI and Sec. V presents the evolution of bright and dark envelope solitons as well as rogons. Finally, Sec. VI is devoted to summarize and conclude the results.

II. FLUID MODEL

We consider the nonlinear propagation of IAWs in a collisionless unmagnetized plasma consisting of q -nonextensive distributed cool and hot electrons (to be denoted, respectively, by the subscripts c and h) and singly charged inertial positive ions (with charge e and mass m_i). At equilibrium, the overall charge neutrality condition reads $n_{h0}/n_{i0} = 1 - f$, where $f = n_{c0}/n_{i0}$ is the ratio between the cool electron and the ion number densities. The dynamics of ion-acoustic perturbations is governed by the following set of normalized fluid equations

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = f n_c + (1 - f) n_h - n_i, \quad (3)$$

where, n_j is the number density of j -species particles ($j = c, h, i$, respectively, stand for cool electrons, hot electrons and positive ions) normalized by their equilibrium

value n_{j0} and u_i is the ion-fluid velocity normalized by the ion-acoustic speed $C_s = \sqrt{k_B T_c / m_i}$ with k_B denoting the Boltzmann constant. Also, ϕ is the electrostatic wave potential normalized by $k_B T_c / e$. Furthermore, the space and the time variables are normalized by the Debye length $\lambda_D = (k_B T_c / 4\pi n_{i0} e^2)^{1/2}$ and the ion plasma period $\omega_{pi}^{-1} = (4\pi n_{i0} e^2 / m_i)^{-1/2}$ respectively. In Eq. (5), the number densities n_c and n_h for cool and hot electrons ($j = c, h$) are given by the following q -nonextensive distributions [21]

$$n_j = n_{j0} \left[1 + \frac{e\phi(q_j - 1)}{k_B T_j} \right]^{(1/2)+1/(q_j-1)}, \quad (4)$$

where q_c (q_h) is a nonextensive parameter for cool (hot) electrons, measuring deviation from the Maxwellian distribution with $q_j \rightarrow 1$.

In the propagation of small but finite amplitude IAWs, one must have $|e\phi/k_B T_c| \ll 1$. Also, as mentioned in the introduction that a q distribution with $q < 1$ is strongly suggested for real plasma systems or superthermal plasmas. So, we take $q_{c,h} < 1$. Thus, Eq. (4) can be binomially expanded (retaining terms up to ϕ^3) to obtain from Eq. (3) the following:

$$\frac{\partial^2 \phi}{\partial x^2} = 1 + (c_1 + d_1)\phi + (c_2 + d_2)\phi^2 + (c_3 + d_3)\phi^3 - n_i, \quad (5)$$

where the coefficients c_j and d_j ($j = 1, 2, 3$) are all positive and given by

$$c_1 = \frac{1}{2}f(q_c + 1), \quad c_2 = \frac{1}{8}f(q_c + 1)(3 - q_c),$$

$$c_3 = \frac{1}{48}f(q_c + 1)(3 - q_c)(5 - 3q_c), \quad d_1 = \frac{1}{2}(1 - f)\beta(q_h + 1),$$

$$d_2 = \frac{1}{8}(1 - f)\beta^2(q_h + 1)(3 - q_h), \quad (6)$$

$$d_3 = \frac{1}{48}(1 - f)\beta^3(q_h + 1)(3 - q_h)(5 - 3q_h).$$

where β ($= T_c/T_h$) is the ratio of cool to hot electron temperatures.

III. DERIVATION OF NLS EQUATION: PERTURBATIVE APPROACH

We employ a multiple-scale perturbation technique [62, 63] to derive the evolution equation of a slowly varying weakly nonlinear wave amplitude of IAW packets in a q nonextensive plasma. Here, we consider A as the state vector $\{A\} (= n_i, u_i, \phi)$, describing the system's state at a given position x and time t . Small deviations will be considered from the equilibrium state $A^{(0)} = (1, 0, 0)^T$

by taking $A = A^{(0)} + \epsilon A^{(1)} + \epsilon^2 A^{(2)} + \dots = A^{(0)} + \sum_{n=1}^{\infty} \epsilon^n A^{(n)}$, where $\epsilon \ll 1$ is a small positive parameter measuring the weakness of the wave amplitude. The wave amplitude is thus allowed to depend on the stretched (slow) coordinates of space and time as $X_n = \sum_n \epsilon^n x$ and $T_n = \sum_n \epsilon^n t$, respectively, where $n = 1, 2, 3, \dots$ (viz. $X_1 = \epsilon x$, $X_2 = \epsilon^2 x$, and so forth; same for time), distinguished from the (fast) carrier variables x ($\equiv X_0$) and t ($\equiv T_0$). All the perturbed states depend on the fast scales via the phase $\theta_1 = kx - \omega t$ only (ω , k are the wave frequency and wave number respectively), whereas the slow scales only enter the l th harmonic amplitude $A_l^{(n)}$. Thus, $A^n = \sum_{l=-\infty}^{\infty} A_l^{(n)}(X, T) e^{il(kx - \omega t)}$ in which the reality condition $A_{-l}^{(n)} = A_l^{(n)*}$ is met by all the state variables.

In what follows, we substitute the above expansion into Eqs. (1), (2) and (5) and equate coefficients of different powers of ϵ . Thus, equating the coefficients of ϵ for $n = 1$, $l = 1$, one obtains

$$-i\omega n_1^{(1)} + iku_1^{(1)} = 0, \quad (7)$$

$$-i\omega u_1^{(1)} + ik\phi_1^{(1)} = 0, \quad (8)$$

$$n_1^{(1)} - (k^2 + c_1 + d_1)\phi_1^{(1)} = 0. \quad (9)$$

The solutions for the first harmonics read

$$n_1^{(1)} = (k^2 + c_1 + d_1)\phi_1^{(1)}, \quad (10)$$

$$u_1^{(1)} = \frac{\omega}{k}(k^2 + c_1 + d_1)\phi_1^{(1)} = \frac{\omega}{k}n_1^{(1)}. \quad (11)$$

Thus, eliminating $n_1^{(1)}$ and $\phi_1^{(1)}$ we obtain the following linear dispersion relation for IAWs

$$\omega = k/\sqrt{k^2 + c_1 + d_1}. \quad (12)$$

This clearly shows the dependency of the wave frequency of IAWs on the nonextensive parameters q_c and q_h as well as the density and temperature ratios f and β . It is seen that the wave frequency increases slowly with k , however, it decreases with increasing values of q_j , f and β . We note that since c_1 and d_1 are positive the linear stability is ensured in Eq. (12).

For $n = 2$ and $l = 1$, we obtain a compatibility condition in the form

$$\frac{\partial \phi_1^{(1)}}{\partial T_1} + V_g \frac{\partial \phi_1^{(1)}}{\partial X_1} = 0, \quad (13)$$

where $V_g \equiv \partial\omega/\partial k$ is the group velocity of the IAW packets given by

$$V_g = \frac{c_1 + d_1}{(k^2 + c_1 + d_1)^{3/2}}. \quad (14)$$

Clearly, the group velocity of the wave becomes constant in the long-wavelength limit $k \rightarrow 0$. This implies that in a frame moving with the group velocity V_g , the time derivatives of all physical quantities should vanish and thus one can observe a slow variation of the wave amplitude in the moving frame of reference. From Eq. (14) we also find that the group velocity decreases not only with the wave number k of the carrier wave, but also with increasing values of the plasma parameters q_j ($j = c, h$), f and β .

Next, the expressions for the wave amplitudes corresponding to $n = 2, l = 1$ are obtained as

$$n_1^{(2)} = -2ik \frac{\partial \phi_1^{(1)}}{\partial X_1} \quad (15)$$

$$u_1^{(2)} = -i\omega \frac{\partial \phi_1^{(1)}}{\partial X_1} \quad (16)$$

Similarly, for $n = 2$ and $l = 2$, the evolution equations provide the amplitudes of the second order, second harmonic modes which are proportional to $(\phi_1^{(1)})^2$. The expressions for them are obtained as:

$$n_2^{(2)} = C_1^{(22)} (\phi_1^{(1)})^2, \quad u_2^{(2)} = C_2^{(22)} (\phi_1^{(1)})^2, \quad \phi_2^{(2)} = C_3^{(22)} (\phi_1^{(1)})^2, \quad (17)$$

where the coefficients are

$$C_1^{(22)} = (c_2 + d_2) + (4k^2 + c_1 + d_1)C_3^{(22)},$$

$$C_2^{(22)} = \frac{\omega}{k} [C_1^{(22)} - (k^2 + c_1 + d_1)^2],$$

and

$$C_3^{(22)} = -\frac{(c_2 + d_2)}{3k^2} + \frac{(k^2 + c_1 + d_1)^2}{2k^2}. \quad (18)$$

Now, for the zeroth harmonic modes we consider the expressions for $n = 2, l = 0$ and $n = 3, l = 0$. Thus, we obtain

$$n_0^{(2)} = C_1^{(20)} (\phi_1^{(1)})^2, \quad u_0^{(2)} = C_2^{(20)} (\phi_1^{(1)})^2, \quad \phi_0^{(2)} = C_3^{(20)} (\phi_1^{(1)})^2, \quad (19)$$

where the coefficients are

$$C_1^{(20)} = [(c_1 + d_1)C_3^{(20)} + 2(c_2 + d_2)], \quad (20)$$

$$C_2^{(20)} = -\frac{2\omega}{k} (k^2 + c_1 + d_1)^2 + V_g C_1^{(20)}, \quad (21)$$

$$C_3^{(20)} = \frac{2(c_2 + d_2)V_g^2 - (k^2 + 3(c_1 + d_1))}{1 - (c_1 + d_1)V_g^2}. \quad (22)$$

Finally, for $n = 3, l = 1$, we obtain equations for third order, first harmonic modes in which the coefficients of $\phi_1^{(3)}$ and $\partial \phi_1^{(2)}/\partial X_1$ vanish by the dispersion relation and the group velocity expression. In the reduced equation, we substitute the expressions for second order, zeroth harmonic modes. Thus, we obtain an equation in the form

$$i \left[\frac{\partial \phi_1^{(1)}}{\partial T_2} + V_g \frac{\partial \phi_1^{(1)}}{\partial X_2} \right] + P \frac{\partial^2 \phi_1^{(1)}}{\partial X_1^2} + Q |\phi_1^{(1)}|^2 \phi_1^{(1)} = 0, \quad (23)$$

where the coefficient of dispersion P and the nonlinearity Q are given by

$$P \equiv \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} = -\frac{3}{2} \frac{k(c_1 + d_1)}{(k^2 + c_1 + d_1)^{5/2}}, \quad (24)$$

$$Q = k(k^2 + c_1 + d_1)^{-3/2} \times \left[(c_2 + d_2) (C_3^{(20)} + C_3^{(22)}) + \frac{3}{2}(c_3 + d_3) \right] - \frac{\omega}{2} (C_1^{(20)} + C_1^{(22)}) - k (C_2^{(22)} + C_2^{(20)}). \quad (25)$$

The nature of these coefficients will be discussed in detail in the next section.

The total (first order) solution obtained for the electric potential is

$$\phi \simeq \phi_1^{(1)} = \phi(x, t) e^{i(kx - \omega t)} + \phi^*(x, t) e^{-i(kx - \omega t)}, \quad (26)$$

where the variation of $\phi(x, t)$ is assumed to be slower than that of $\exp[i(kx - \omega t)]$ ($\epsilon = 1$ was formally set here, with the understanding that $|\phi| = |\phi_1^{(1)}| \ll 1$, i.e., remains small). The compatibility condition is now written in the form of a NLS equation for small and slowly varying wave amplitude $\phi(x, t)$ as

$$i \left(\frac{\partial \phi}{\partial t} + V_g \frac{\partial \phi}{\partial x} \right) + P \frac{\partial^2 \phi}{\partial x^2} + Q |\phi|^2 \phi = 0. \quad (27)$$

Performing a Galilean transformation, namely $\xi = x - V_g t$, $\tau = t$, Eq. (27) can be written in its standard form as

$$i \frac{\partial \phi}{\partial \tau} + P \frac{\partial^2 \phi}{\partial \xi^2} + Q |\phi|^2 \phi = 0 \quad (28)$$

where dispersive and nonlinear coefficients P and Q respectively are given by Eqs. (24) and (25). Since P and Q are dependent on various physical parameters viz f , β , q_c and the carrier wave number k , so it is very important to understand the nature of solitary structures from the solutions of Eq.(28) under the influence of such parameters.

IV. MODULATIONAL INSTABILITY

We consider the amplitude modulation of a plane wave solution of Eq. (28) of the form $\phi = \phi_0 e^{-i\Omega_0 \tau}$, where $\Omega_0 = -Q|\phi_0|^2$ is the nonlinear frequency shift with ϕ_0 denoting the potential of the wave pump. We then modulate the wave amplitude with a plane wave perturbation with frequency Ω and wave number K as $\phi = (\phi_0 + \epsilon \phi_1 e^{iK\xi - i\Omega\tau} + c.c.) e^{-i\Omega_0 \tau}$. One thus obtains the following dispersion relation for the modulated IAW packets.

$$\Omega^2 = (PK^2)^2 \left(1 - \frac{K_c^2}{K^2} \right), \quad (29)$$

where $K_c = \sqrt{2|Q/P|}|\phi_0|$ is the critical value of the wave number of modulation. Thus, the MI sets in for $K < K_c$ and the wave will be modulated for $PQ > 0$, or more precisely, for $Q < 0$ since P is always negative. In this case, the perturbations grow exponentially during the propagation of IAW packets. On the other hand, for $K > K_c$, i.e., for $PQ < 0$, or $Q > 0$, the IAWs are said to be stable under the modulation. The growth rate of MI can be obtained from Eq. (29) as

$$\Gamma = |P|K^2 \left(\frac{K_c^2}{K^2} - 1 \right). \quad (30)$$

Clearly, the maximum value Γ_{\max} of Γ is achieved at $K = K_c/\sqrt{2}$ and is given by $\Gamma_{\max} = |Q||\phi_0|^2$.

We note that the coefficients P , Q and hence the critical value K_c as well as the growth rate Γ all depend not only on the plasma parameters q_j , f and β , but also on the carrier wave number k . In the range of values of k , for which the MI is excluded, i.e., $PQ < 0$ or $Q > 0$, the perturbations may develop into dark solitons which represent an electric potential envelope hole with finite or vanishing potential at the origin. Such solitons occur in plasmas under the effects of self-modulation. On the other hand, for $PQ > 0$, i.e., for $Q < 0$, the excitation of bright envelope solitons may be possible. Although, envelope solitons appear to be analytical coincidence via this model, and the phenomena should be clearly distinguished [linear amplitude stability analysis in one case, nonlinear theory of partial differential equations (PDE) in the other], it has been postulated in another context [64] that MI is the first evolutionary stage towards the formation of envelope solitons. The latter in the form of bright type are widely observed in abundance in space plasmas [29, 63, 65].

Inspecting on the expressions for P and Q it is found that P is always negative as $c_1, d_1 > 0$, however, Q can be positive or negative depending on the range of values of k and the parameters q_j , f and β . Now, for $k \ll 1$, P can be written as $P \simeq -p_0 k$, i.e., P tends to vanish as $k \rightarrow 0$. This is expected for an ion-acoustic mode where $\omega \sim k$ and $d^2\omega/dk^2 = 0$. However, Q increases as $Q \simeq q_0/k$ in the limit $k \rightarrow 0$. These are in qualitative agreement with earlier theoretical investigations [29, 66].

Also, in the limit $k \rightarrow 0$, the real quantities p_0 and q_0 are, in fact, both positive, given by

$$p_0 = \frac{3}{2(c_1 + d_1)^{3/2}}, \quad (31)$$

$$q_0 = \frac{1}{12} \left(\frac{1}{c_1 + d_1} \right)^{3/2} [2(c_2 + d_2) - 3(c_1 + d_1)^2]. \quad (32)$$

Thus, we find that the IAW packets with long-wavelength carrier wave modes are always stable under the modulation. In the opposite limit, i.e., for $k \gtrsim 1$, we will see from the numerical investigation that the waves are mostly modulationally unstable.

Since P is always negative, the sign of PQ , for which the stability and instability of the IAW envelopes can be determined, solely depends on the sign of Q . So, we numerically investigate the nonlinear coefficient Q for different values of the parameters q_j , f and β . Typical behaviors of Q with respect to k are shown in Fig. 1. While the upper panel shows the properties of Q for different values of β and f , the lower one shows the same with the variations of the nonextensive parameters q_c and q_h . It is seen that whatever be the values of the plasma parameters, Q is always positive in the limit $k \rightarrow 0$ (i.e., at long scale perturbations of the carrier modes) as mentioned above, whereas it becomes negative for $k > 1$ as well as for some intermediate values of k in $0 < k < 1$, i.e. at small and medium scale perturbations. It is also evident that Q becomes maximum (hence the maximum growth rate of instability) in the limit $k \rightarrow 0$ as mentioned before except for the case where q_c, q_h increase from some positive values to the value 1 (see the dotted and dash-dotted lines in the lower panel) in which Q can be maximum at an intermediate value of k in the interval $0 < k < 1$. Typical behavior of Q (in the case of $q_c \neq q_h < 1$) shows that initially it drops down from its maximum value in the domain $0 < k \lesssim 0.1$ and then starts increasing in a small interval of $k \gtrsim 0.1$, and again it decreases faster the larger is the wave number k of carrier modes. However, as the values of the nonextensive parameters q_j increase and approach a value 1 (i.e., one approaches the Maxwellian distribution of electrons) Q initially increases in a small interval $0 < k < 0.5$, and then it decreases faster with increasing values of k . From Fig. 1, we also observe that as the parameters β and f get enhanced, the values of Q decrease, and the ranges of values of k for which $Q < 0$ increase (See the upper panel). The same are true for increasing values of q_c (See the lower panel), but decreasing values of q_h [except in the range where $k \rightarrow 0$ in which the value of Q increases with decreasing values of q_h , however, the range of k for which $Q < 0$ remains unaltered (See the dotted and dash-dotted lines the lower panel)]. Thus, from Fig. 1 we conclude that the IAW packets with short-wavelength carrier modes ($k > 1$) are always unstable with $Q < 0$, $P < 0$ under the amplitude modulation with plane wave perturbation, it can also be unstable at the medium scale with

k lying in $0.5 \lesssim k < 1$ or more generally in $0 < k < 1$. In these cases the modulational instability leads to the formation of bright envelope solitons or highly energetic rogue waves (rogons). On the other hand, the IAW packets can be stable ($Q > 0$, $P < 0$) for long-wavelength carrier modes with $k \ll 1$ leading to the formation of dark envelope solitons.

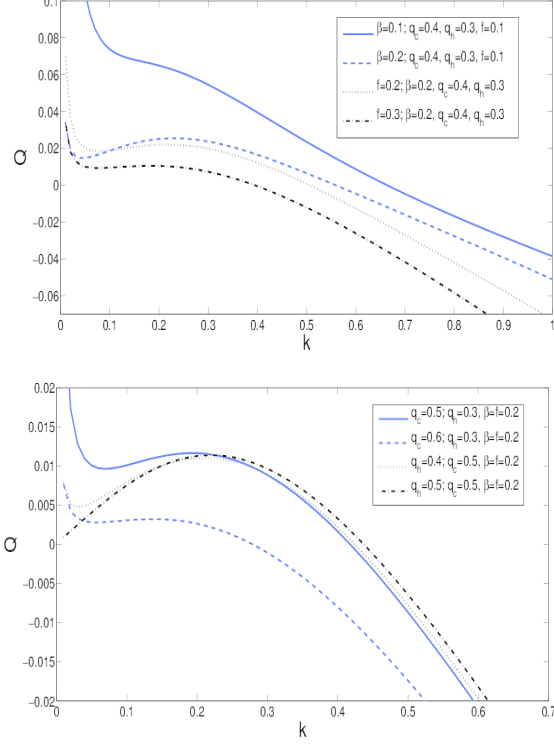


FIG. 1. Plot of the nonlinear coefficient Q of the NLS equation (28) versus the carrier wave number k is shown for different values of the parameters β , f , q_c and q_h as in the upper and lower panels. Since P is always negative, the stable ($PQ < 0$) and unstable ($PQ > 0$) regions are corresponding to $Q > 0$ and $Q < 0$ respectively.

In what follows, we numerically investigate the growth rate of instability Γ for different values of the plasma parameters q_j , f and β as in Fig. 1. The behaviors of Γ are shown in Fig. 2. The critical values K_c of the wave number of modulation K below which the MI sets in can easily be calculated. It is found that as the parameters β , f and q_c increase, the values of K_c increase, however, the same decreases with increasing values of β_h . This is expected as $K_c \sim Q/P$. While the upper panel shows the variation with β and f , the lower panel exhibits growth rates for q_c and q_h . We find that a small increase of the values of β , f (upper panel) and q_c (lower panel), leads to a significantly higher growth rate of instability with cut-offs at higher values of K . However, as q_h increases, the growth rate decreases (but not significantly lower) with cutoff at lower K . Thus, we find that the growth rate of instability can be suppressed in plasmas with higher

temperature of hot electrons than the cold electrons, the lower concentration of cold electrons than ions and more (less) superthermal cold (hot) electrons.

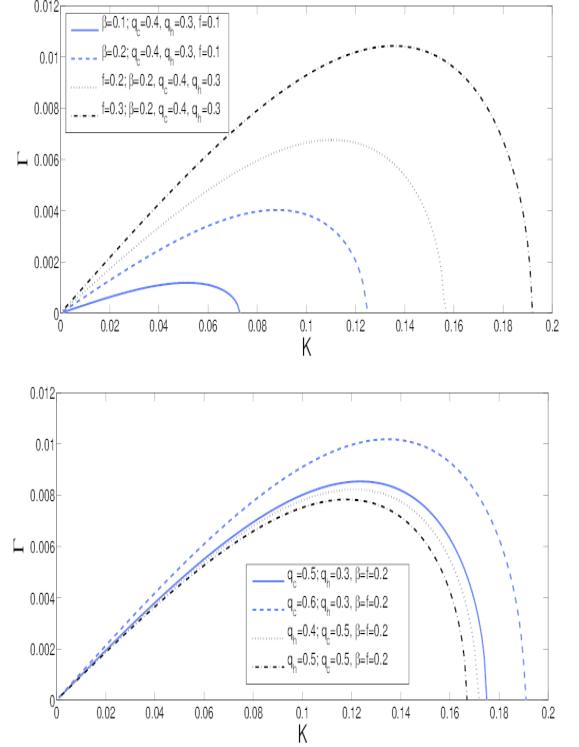


FIG. 2. The growth rate of instability Γ is shown against the wave number of modulation K with the same parameters β , f and q_j as in Fig. 1. The other parameter values are $k = 0.7$ and $\phi_0 = 0.5$. It is seen that the lower (higher) values of q_c , f and β (q_h) are to suppress Γ with cutoffs at lower (higher) wave numbers of modulation.

V. ENVELOPE SOLITONS AND ROGONS

In the previous section we have mentioned that when $PQ > 0$, the MI of IAW packets can give rise to the formation of bright envelope solitons due to energy localization. In this case, an exact analytic (soliton) solution of the NLS equation (28) can be obtained by considering $\phi = \sqrt{\psi} \exp(i\theta)$, where ψ and θ are real functions [see for details, e.g., Refs. 63, 67, 68 as

$$\begin{aligned} \psi &= \psi_{b0} \operatorname{sech}^2 \left(\frac{\xi - U\tau}{W_b} \right), \\ \theta &= \frac{1}{2P} \left[U\xi + \left(\Omega_0 - \frac{U^2}{2} \right) \tau \right]. \end{aligned} \quad (33)$$

This solution, in fact, represents a localized pulse traveling at a speed U and oscillating at a frequency Ω_0 at rest. The width W_b and the constant amplitude ψ_{b0} of the pulse are related to $W_b = \sqrt{2P/Q\psi_{b0}}$. It follows that

for a constant amplitude, as the parameters β , f and q_c increase, the width of the envelope soliton decreases, however, the same increases with increasing values of β_h .

On the other hand, for $PQ < 0$, the IAW packets are modulationally stable which may propagate in the form of dark-envelope solitons characterized by a depression of the wave potential around $\xi = 0$. Typical form of this solution of Eq. (28) is given by

$$\begin{aligned}\psi &= \psi_{d0} \tanh^2 \left(\frac{\xi - V\tau}{W_d} \right), \\ \theta &= \frac{1}{2P} \left[V\xi - \left(\frac{V^2}{2} - 2PQ\psi_{d0} \right) \tau \right].\end{aligned}\quad (34)$$

This represents a localized region of hole (void) traveling at a speed V . The pulse width W_d depends on the constant amplitude ψ_{d0} as $W_d = \sqrt{2|P/Q|/\psi_{d0}}$. Clearly, for a constant amplitude ψ_{d0} the width of the dark envelope solitons have the similar properties as the bright solitons with the variations of the plasma parameters. Typical profiles of the bright (at $k = 0.7, 1.5$) and dark (at $k = 0.1$) envelope solitons are shown in Fig. 3 for the same parameter values as for the dotted line in the lower panel of Fig. 1. It is seen that while the bright envelope solitons exist at small and medium scales of perturbations, the dark envelope solitons can exist only at the large scale $k \ll 1$.

In the ranges of values of k for which the IAW packet becomes unstable ($PQ > 0$), Eq. (28) can admit highly energetic rogue wave solutions. These rogue waves or rogons, in which a significant amount of energy is concentrated in a relatively small area in space and time, are generated due to the MI of the coherent IAW packets in the limit of infinite wave modulation period. In particular, they significantly amplify the carrier wave amplitudes, and hence increase the nonlinearity during the evolution of the wave packets. Now, Eq. (28) can be rewritten as

$$i \frac{\partial \phi}{\partial \tilde{\tau}} + \frac{1}{2} \frac{\partial^2 \phi}{\partial \tilde{\xi}^2} + |\phi|^2 \phi = 0, \quad (35)$$

where $\tilde{\tau} = Q\tau$ and $\tilde{\xi} = \sqrt{Q/2P}\xi$. The rogue wave solution of Eq. (35) that is located on a non-zero background and localized in both space and time can be obtained as [48]

$$\phi_n(\tilde{\xi}, \tilde{\tau}) = \left[(-1)^n + \frac{G_n(\tilde{\xi}, \tilde{\tau}) + iH_n(\tilde{\xi}, \tilde{\tau})}{D_n(\tilde{\xi}, \tilde{\tau})} \right] \exp(i\tilde{\tau}), \quad (36)$$

where G_n , H_n and D_n ($\neq 0$) are some polynomial functions of $\tilde{\xi}$ and $\tilde{\tau}$, and $n = 1, 2, 3, \dots$, denotes the order of the solution. The first-order rogon solution is obtained by considering $n = 1$ and it corresponds to the Peregrine soliton [42] in which $G_1 = 4$, $H_1 = 8\tilde{\tau}$ and $D_1 = 1 + 4\tilde{\xi}^2 + 4\tilde{\tau}^2$. However, superposition of two first-order rogue waves can lead to the generation

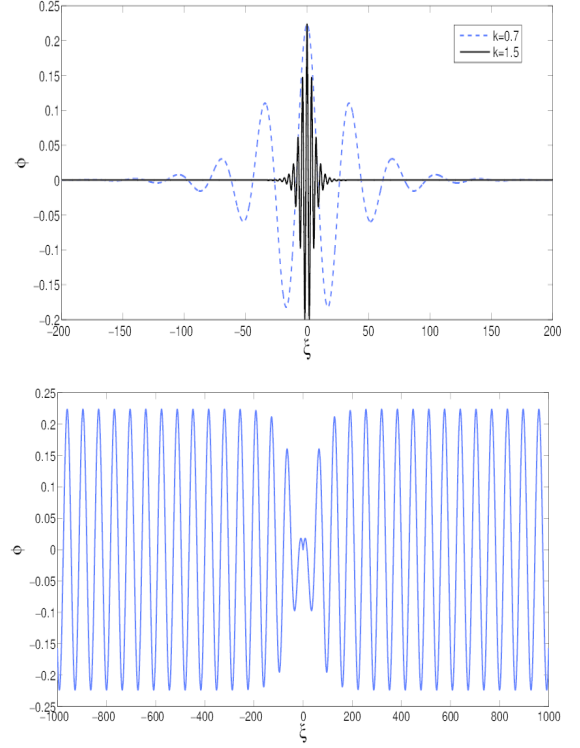


FIG. 3. Plots of the bright (upper panel) and dark (lower panel) envelope solitons at $\tau = 0$ given by Eqs. (33) and (34) respectively. It is seen that while the bright envelope solitons exist both at the small (e.g., $k = 1.5$) and medium (e.g., $k = 0.7$) scales, the dark solitons exist only at the large scales (e.g., $k = 0.1$). The parameter values of q_j , f and β are the same as for the dotted line in the lower panel of Fig. 1. The other values are taken as $\psi_{b0} = \psi_{d0} = 0.05$, $U = 0.2$ and $\Omega_0 = 0.5$.

of another highly energetic rogue waves with higher amplitudes. The analytic form of these waves have been recently obtained by Akhmediev et al. [47] using the deformed Darboux transformation approach. This second order ($n = 2$) rogon solution has the same form as Eq. (36) where the polynomials G_2 , H_2 and D_2 are given by

$$G_2 = - \left(\tilde{\xi}^2 + \tilde{\tau}^2 + \frac{3}{4} \right) \left(\tilde{\xi}^2 + 5\tilde{\tau}^2 + \frac{3}{4} \right) + \frac{3}{4}, \quad (37)$$

$$H_2 = \tilde{\tau} \left[3\tilde{\xi}^2 - \tilde{\tau}^2 - 2 \left(\tilde{\xi}^2 + \tilde{\tau}^2 \right)^2 - \frac{15}{8} \right], \quad (38)$$

$$\begin{aligned}D_2 &= \frac{1}{3} \left(\tilde{\xi}^2 + \tilde{\tau}^2 \right)^3 + \frac{1}{4} \left(\tilde{\xi}^2 - 3\tilde{\tau}^2 \right)^2 \\ &\quad + \frac{3}{64} \left(12\tilde{\xi}^2 + 44\tilde{\tau}^2 + 1 \right).\end{aligned}\quad (39)$$

The profiles of these rogon solutions given by Eq. (36) for $n = 1, 2$ are shown in Fig. 4. We find that for $k < 1$ ($k > 1$), the localization occurs at large (small) ξ . The

first-order Peregrine soliton has been recently observed experimentally in plasmas [45]. However, the second-order rogon solution, which has been observed in water waves [69], is yet to observe in plasmas. The amplification factor of the amplitude of the n -th order rational solution [Eq. (36)] at $\xi = \tilde{\tau} = 0$ is, in general, of $2n + 1$. Hence, localized IAWs that are modeled by the higher-order breather solutions can also cause the formation of super rogue waves. In order to investigate the properties

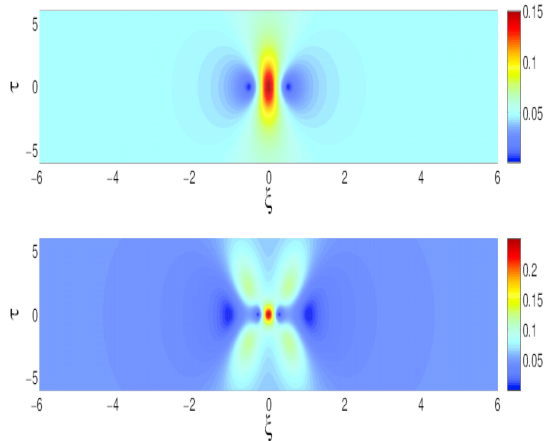


FIG. 4. Profiles (contour plots) of the first ($n = 1$) and second ($n = 2$) order rogon solutions given by Eq. (36) of the NLS equation (35) are shown in the upper and lower panels respectively. The parameter values are $q_c = 0.4$, $q_h = 0.3$, $f = 0.2$, $\beta = 0.6$ and $k = 1.5$.

of these rogue wave solutions more clearly for different values of the parameters q_j , f and β , we plot the solutions at $\tau = 0$. These are shown in Fig. 5. We find that while the amplitudes of both the first and second order rogons remain the same, their widths decrease with increasing values of q_c (i.e., lack of superthermal cold electrons), f and the temperature ratio β . However, no significant change of the width is found with a small increase of β_h for both the rogons.

VI. SUMMARY AND CONCLUSION

In this work, we have investigated the amplitude modulation of ion-acoustic wave (IAW) packets in an unmagnetized multi-component plasma consisting of singly charged positive ions and two-temperature electrons featuring nonextensive distributions. Using the multiple-scale perturbation method, a nonlinear Schrödinger (NLS) equation is derived which governs the evolution of IAW envelopes. This equation is then used to study the modulational instability (MI) of a Stokes' wave train to a small longitudinal perturbation. Different stable and unstable regions of modulation in the range of the carrier wave number k are found out with the variations of the plasma parameters, namely, the nonextensive indices q_c

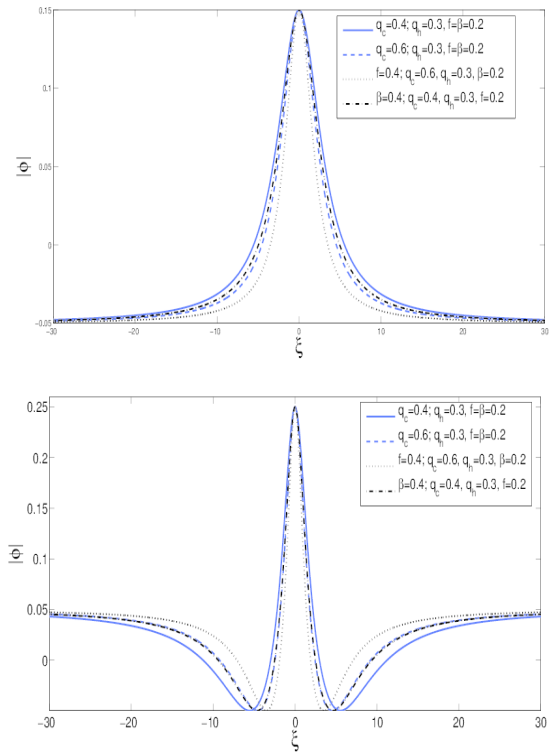


FIG. 5. Properties of the first ($n = 1$) and second ($n = 2$) order rogon solutions at $\tau = 0$ given by Eq. (36) of the NLS equation (35) are shown in the upper and lower panels respectively for different values of the parameters as in the figure. For the second order rogons an increase of $q_c = 0.6$ or $\beta = 0.4$ effectively gives almost the same results (See the dashed and dash-dotted lines).

and q_h , the ratio of cool electron to ion number densities f as well as the cool to hot electron temperature ratio β . It follows that the modulated IAW packets can propagate in the form of bright envelope solitons or highly energetic rogue waves (rogons) due to MI as well as dark envelope solitons where the MI is excluded. The growth rate of instability together with the exact analytical solutions of the NLS equation in the form of bright and dark envelope solitons as well as rogons are also obtained and their properties are examined numerically with the plasma parameters. The main results can be summarized as follows:

- The fluid model governing the dynamics of small-amplitude IAWs in multi-component electron-ion plasmas is valid for the plasma parameters satisfying $0 < f, \beta, q_c, q_h < 1$, i.e., plasmas with more superthermal electrons [compared to the Maxwellian ones ($q \rightarrow 1$)] and lower concentration of cold electrons than the positive ions.
- The group velocity dispersive coefficient P of the NLS equation is always negative irrespective of the values of k and the plasma parameters. However, the nonlinear coefficient Q is always negative for

$k \gtrsim 1$ (and any values of the parameters with $0 < f, \beta, q_c, q_h < 1$) and can become positive in the limit $k \rightarrow 0$ as well as in a sub interval of $0 < k < 1$ depending on the plasma parameters. Thus, the modulated IAW packets are always stable (unstable) with long-wavelength (short-wavelength) perturbations (carrier modes) giving rise the formation of dark envelope solitons (bright envelope solitons or highly energetic rogons). The wave envelope can even be unstable at a medium scale of perturbation with $0 < k \lesssim 1$.

- As β and f increase, the value of Q decreases (hence decreasing the maximum growth rate of instability) and the range of values of k for which $Q < 0$ (hence the MI) increases. The same is true for increasing values of q_c , but decreasing values of q_h except for some region with $k \rightarrow 0$.
- As the parameters β , f and q_c increase, the values of the critical wave number K_c below which the MI sets in increase, however, the same decreases with increasing values of β_h . It turns out that the growth rate of instability can be suppressed in superthermal plasmas with higher temperature of hot electrons than the cool electrons, the lower concentration of cold electrons than ions and more (less) superthermal cool (hot) electrons.
- In the range of values of k where $PQ > 0$, bright envelope solitons are shown to exist both at the

small ($k > 1$) and medium ($k \lesssim 1$) scale of perturbations. However, dark solitons exist only at large scales ($k \ll 1$).

- The MI of IAW envelopes can also lead to the generation of rogue waves in which a significant amount of wave energy is concentrated in relatively a small area in space and time. Typical profiles of the first and second-order rogons are presented with respect to finite space and time. It is found that at an equilibrium state, while the amplitudes of both the first and second order rogons remain the same, their widths decrease with increasing values of q_c (i.e., lack of superthermal cold electrons), f and the temperature ratio β . However, no significant change of the width is found with a small increase of β_h for both the rogons.

To conclude, the findings of the present investigation should be relevant both in space and astrophysical plasma environments where two-temperature electrons are the main constituents together with ions. We also propose that an experiment should be done to observe the role of higher order effects for the formation of highly energetic ion-acoustic rogue waves (super rogue waves).

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